Groundwater

Determination of Hydraulic Conductivity from Grain-Size Distribution for Different Depositional Environments

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Abstract

Over 400 unlithified sediment samples were collected from four different depositional environments in global locations and the grain-size distribution, porosity, and hydraulic conductivity were measured using standard methods. The measured hydraulic conductivity values were then compared to values calculated using 20 different empirical equations (e.g., Hazen, Carman-Kozeny) commonly used to estimate hydraulic conductivity from grain-size distribution. It was found that most of the hydraulic conductivity values estimated from the empirical equations correlated very poorly to the measured hydraulic conductivity values with errors ranging to over 500%. To improve the empirical estimation methodology, the samples were grouped by depositional environment and subdivided into subgroups based on lithology and mud percentage. The empirical methods were then analyzed to assess which methods best estimated the measured values. Modifications of the empirical equations, including changes to special coefficients and addition of offsets, were made to produce modified equations that considerably improve the hydraulic conductivity estimates from grain size data for beach, dune, offshore marine, and river sediments. Estimated hydraulic conductivity errors were reduced to 6 to 7.1 m/day for the beach subgroups, 3.4 to 7.1 m/day for dune subgroups, and 2.2 to 11 m/day for offshore sediments subgroups. Improvements were made for river environments, but still produced high errors between 13 and 23 m/day.

Introduction

Hydraulic conductivity is related to the grain-size distribution (sorting), the effective porosity, grain shape, and packing in sediments. It has become increasingly important to be able to accurately estimate the hydraulic conductivity of unlithified sediments to allow predesign screening of marine and terrestrial sediment types in the engineering design of natural filtration projects, such as bank filtration, rapid infiltration basins, aquifer recharge and recovery systems, seabed and beach galleries used for intakes to desalination plants, and for various other hydrogeologic investigations (Maliva and Missimer 2010, 2012; Missimer 2009; Sesler and Missimer in press). Also, soil columns used to assess the removal of pathogens, algae, and trace organic contaminants need to be characterized to assure that similar hydraulic properties are being used in laboratory experiments and field tests (Lewis and Sjöstrom 2010).

An extensive research literature exists concerning the estimation of hydraulic conductivity of unlithified sediment from grain-size distribution data (Kasenow 2002; Vukovic and Soro 1992). The methods can be separated into those that are based on analogies to pipe or capillary flow (Carman 1937; Collins 1961; Fair and Hatch 1933; Kozeny 1927), and empirical relationships between grain-size distribution and hydraulic conductivity (or permeability) (Alyamani and Sen 1993; Barr 2001; Berg 1970; Hazen 1911; Koltermann and Gorelick 1995; Krumbein and Monk, 1943; Morrow et al. 1969; Shepard, 1989; Slichter 1899; Terzaghi 1925; Uma et al. 1989). Most of these estimation methods have been validated using a very limited number of actual measurements of hydraulic conductivity. Most of the empirical relationships involve use of specific aspects of the size distribution, such as the d_{10} value. Krumbein and Monk (1943) used a statistical approach for the determination of the hydraulic conductivity using a transformation of the grain-size distribution to a logarithmic frequency distribution and by incorporating various moments into an empirical equation. Another empirical method proposed by Sperry and Peirce

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(1995) uses a coefficient for sphericity by measurement of the angle of repose. However, this method can only be applied to relatively uniform sediments, because it is not practical for making measurements in poorly sorted sediments containing pebbles or cobbles.

A number of research papers have been published that compare measured hydraulic conductivity measurements to values estimated from grain-size data using empirical equations applied to various aquifers and depositional environments with emphasis on streambed sediments (Alyamani and Sen 1993; Bedinger 1961; Cheng and Chen 2007; Cheong et al. 2008; Folk and Ward 1957; Hrbar and Potter, 1969; Keech and Rosene 1964; Krumbein and Monk 1943; Masch and Denny 1966; Mavis and Wilsey 1936). Also, the hydraulic conductivity of engineered filter sands and artificial mixes of gravel, sand, and clay have been evaluated using both measured and estimated methods (Hazen 1892, 1911; Hulbert and Feban, 1933; King 1899; Schriever 1930). Shepard (1989) analyzed a number of hydraulic conductivity measurements and corresponding grain-size distributions from the literature. He assessed relationships for various depositional environments using regression plots, but did not develop any new equations. While these methods and data assessments are somewhat useful, the models and equations require adjustments for the general grain-size distribution characteristics of the specific sediment types corresponding to the transport mode and depositional environment in which the sediment is found.

The primary purposes of this research are: (1) to assess the accuracy of 20 different empirical methods commonly used to estimate hydraulic conductivity from grain-size distribution data by statistically comparing actual hydraulic conductivity measurements made on 431 sediment samples with the estimated values, (2) to identify which of the 20 empirical methods produce the best hydraulic conductivity estimates for each depositional environment and sub-environment, (3) to explore the relationships between statistical moments of the size distributions of sediment samples and potential improvements to the empirical equations that can be applied for hydraulic conductivity estimation, and (4) to make modifications to the existing empirical equations that improve the accuracy of hydraulic conductivity estimation for specific depositional environments.

Methods

Sediment Samples

Four different depositional environments were chosen for analysis, which include beach, dune, offshore shallow marine, and river sediments with samples collected from perennial and ephemeral (wadis) types. Subsets of sediment types within these general environments were also analyzed based on differences in mud content and lithology (siliciclastic vs. carbonate). A total of 431 samples were chosen for analysis. These samples were collected from global locations and covered siliciclastic, carbonate, and mixed sediment types (Table S1 shows all locations). The samples were typically collected from the upper 5 to 10 cm of the depositional environment. In most cases, a global positioning system (GPS) reading was obtained to describe the precise location of the sample.

Grain-Size Analyses

The grain-size characteristics of each sample were determined using the standard sieving technique as described by Tanner and Balsillie (1995) and the American Society of Testing and Materials (1995). Prior to analysis each sample was carefully prepared by removal of any organic debris and leaching of salt. Removal of salt from the samples was achieved by flushing with fresh water within a container and decanting, while being careful to not remove any naturally occurring mud. Each cleaned bulk sample was dried in an oven at 80 °C for about 2 h. Between 60 and 70 g of sample was collected from the bulk sample and analyzed with the exception of river or poorly sorted samples having a wide range of grain sizes. Because of the large variation in the grain-size characteristics, particularly in wadi sediments, a larger sample of up to 400 g was analyzed. In all cases, 34 sieves were used in the separation process with the size increment corresponding to 0.25-phi units (phi $= -\log_2$ [mm]; Krumbein 1934). The sieving process used a RoTap machine and the time was set to 30 min for the process as recommended by Tanner and Balsillie (1995). The sediment quantity was weighed in each screen to an accuracy of 0.01 g. Each sample was weighed before the sieving process and the sample weights from the sieve increments were summed to assess any sediment loss or calculation errors.

Laboratory Porosity Measurements

Estimation of bulk porosity is generally problematical when assessing the hydraulic properties of unlithified samples. In order to obtain an estimate of total porosity of each sample, a 250 (cc), a 500 (cc), or 1000 mL (1000 cc) graduated cylinder was first filled partially with a known volume of water. Sediment was carefully added to the cylinder and allowed to settle and compact. Additional water and sediment were added to the column. Care was taken to not allow air entrapment within the saturated column. The column was compacted slightly by tapping the side of the cylinder with a rubber mallet to approximate natural system packing conditions near surface. The volume of sediment was determined along with the volume of water added. When the water level in the cylinder rose above the surface of the sediment, a correction was made to the water volume added (subtraction). Then, the estimated porosity was determined by dividing the volume of water by the volume of sediment.

Permeameter Measurements

The hydraulic conductivity of the sediment samples was determined using a standard constant head permeameter based on the methodology described by Wenzel (1942) and following the American Society for Testing and Materials (2006) standard D2434-68. Sediment samples were added to the permeameter chamber in a wet state in most cases to avoid stratification and were compacted by tapping the exterior of the permeameter cylinder with a rubber mallet. A minimum of 5 cm of column height of sediment was placed in the permeameter for measurement. A greater column height was necessary when the samples contained significant quantities of mud, because if the height was less than 5 cm, some of the fine sediments tend to be washed from the permeameter during analysis, thereby increasing the value of hydraulic conductivity obtained. Great care was taken to remove trapped air from the sediment before the measurements were conducted by placing the sediment in the cylinder wet or saturating a dry sample very slowly from the bottom to the top. Flow through the permeameter was from the bottom to the top and was allowed to reach an equilibrium state before time measurements were recorded. Three to five measurements were made of the time to fill a 500 mL cylinder for most samples. A 1000 mL cylinder was used for samples with a high hydraulic conductivity to reduce the error in time measurement. The temperature of the water was also measured during each analysis. The hydraulic conductivity (cm/s) was determined by multiplying the known volume of the collection cylinder (cm^3) by the sediment thickness (cm) and then dividing it by the area of the permeameter (cm^2) times the time required to fill the cylinder (s) times the fixed head (cm).

Empirical Methods Used to Estimate Hydraulic Conductivity from Grain-Size Distribution

Performance of calculations to estimate the hydraulic conductivity from grain-size distribution for numerous samples can be a quite tedious and laborious process and selection of the method used can be critical to the overall accuracy of the exercise. A series of 20 empirical methods (Table 1) were used to estimate the hydraulic conductivity from the grain-size distribution by employing a spreadsheet program. A second program, written in MATLAB®, was developed to provide both an estimation of hydraulic conductivity and to perform statistical analyses of the predicted vs. measured hydraulic conductivities. Descriptions of these programs, information on their use, and the actual programs are included in an appendix to this paper and are published online (Appendix S1).

There is a need to know the statistical moments of the grain-size distribution, especially the mean grain diameter in millimeters, when using grain-size data to design a filter or sediment column. Additionally, sedimentary geologists need to understand the statistical properties of naturally occurring sediments using the phi scale because of the log-normal grain-size distribution of sediments in general (Krumbein 1934) and some of the empirical equations use these moments as variables. For this reason, the phi statistical moments are also calculated in the spreadsheet, based on a fundamental statistical approach described in Tanner and Balsillie (1995). Within the programs the hydraulic conductivity measurements made in the laboratory were normalized to an equivalent water temperature of 25 °C and the default for all grain-size methods calculated hydraulic conductivities was also set at 25 °C, so that the statistical analyses would be based on compatible data. The normalization process required that the hydraulic conductivity values had to be converted into permeability and then the kinematic viscosity had to be adjusted for the temperature. The permeability values were then converted back to hydraulic conductivity.

Statistical Methods Applied to Data

A statistical analysis of the estimated values using the empirical equations vs. measured values for hydraulic conductivity was conducted. An assessment was made concerning which specific empirical methods are best applied to sediments within the various depositional environments and subsets.

The different sets of samples, classified in terms of depositional environments and sub-environments, were first analyzed to obtain a range of values for their physical properties, such as d_{10} and mud percentage (grain size below 0.0625 mm). A range was set at three times the standard deviation around the mean of each property. Samples with at least one physical property outside of this range were considered to be outliers.

Samples from each dataset minus the outliers were subjected to a simple regression analysis to assess the relationship between the measured and estimated hydraulic conductivity values obtained from the appropriate empirical methods. The result is a linear equation for the measured hydraulic conductivity as a function of the estimated hydraulic conductivity for each chosen empirical method. The beta coefficients within the empirical equations were then adjusted in order to obtain a linear equation with a unit slope. By doing so, the measured hydraulic conductivity is represented by an offset of the linear fit of the estimated hydraulic conductivity. The efficacy of the methods can be compared in this way by analyzing the correlation coefficients of the linear fit. The p-value was used to measure the probability of obtaining a statistical distribution at least as extreme as the data set (measured vs. estimated hydraulic conductivities) assuming there is no relationship between the two variables. A value of less than 0.01 is considered to be a significant correlation. Also, the p-value is reported as $-\ln(p)$ for convenience. Therefore, a value above 4.6 is considered to be a significant correlation. The R^2 value was used as a measure of the linearity of the relationship, or the distribution of the values around the linear fit.

The modified empirical equations were placed in a separate spreadsheet program which contains the classifications of the depositional environments with the associated modified and improved empirical methods for use in estimation (see Appendix S1 for program and user guide).

Table 1
Methods Used to Estimate Hydraulic Conductivity from Grain Size Data (in m/day Unless Stated)

Method	Equation	Variable and Unit Definition	β	Use
Alyamani and Sen (1993)	$K\left[\frac{m}{d}\right] = \beta\left[I_o + 0.025\left(d_{50} - d_{10}\right)\right]^2$	I_o is the intercept in mm of the line formed by d_{50} [mm] and d_{10} [mm] with the grain-size axis	1300	Well-distributed sample
Barr (2000)	$K\left[\frac{m}{s}\right] = \beta\frac{\rho g}{\mu}nm^2$	m is the hydraulic radius	1/5	$1 < C_s < 1.35$
	$m = \frac{n}{S}$	S is the surface area		
	$S = C_s S_o(1 - n)$	C_s is a surface area adjusting parameter		
	$S_o = \sum_i S_{oi}$	S_{oi} is the surface area per unit mass of solid material		
	$S_{oi} = \frac{3}{r_i}\cdot wf_i$	r is the radius of the sphere representing the grain (sieve size), in meters wf_i is the weight fraction retained in sieve i		
Beyer (1964)	$K\left[\frac{m}{s}\right] = \beta\frac{g}{v}\log\frac{500}{C}d_{10}^2$	C is the coefficient of uniformity	6×10^{-4}	$0.06\text{ mm} < d_{10} < 0.6\text{ mm}$
	$C = \frac{d_{60}}{d_{10}}$			$1 < C < 20$
Chapuis et al. (2005)	$K\left[\frac{cm}{s}\right] = \beta\left(\frac{d_{10}^2 e^3}{1+e}\right)^{0.7825}$	e is the void ratio	2.4622	$0.03\text{ mm} < d_{10} < 3\text{ mm}$
	$e = \frac{n}{1-n}$	d_{10} is in mm		$0.3 < e < 0.7$
Fair and Hatch (1933)	$K\left[\frac{m}{s}\right] = \beta\frac{\rho g}{\mu}\frac{n^3}{(1-n)^2}\frac{1}{m\left(\frac{\theta}{100}\sum_i\frac{P_i}{d_{mi}}\right)}$	m is a packing factor	1	$m = 5$
	$P_i = 100\cdot wf_i$	θ is a sand shape factor		$6 < \theta < 7.7$, spherical to angular respectively
	$d_{mi} = \sqrt{ds_i\cdot ds_{i+1}}$	P is the percentage of sand held between adjacent sieves d_m is the geometric mean ds_i is the size of the i sieve		
Harleman et al. (1963)	$K\left[\frac{m}{s}\right] = \frac{\rho g}{\mu}d_{10}^2$		6.54×10^{-4}	

Table 1
Continued

Method	Equation	Variable and Unit Definition	β	Use
Hazen-original (1892)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}\left[1+10\ (n-0.26)\right]d_{10}^2$		6×10^{-4}	0.1 mm $< d_{10} <$ 3 mm C < 5
Hazen-new (modified)	$K\left[\frac{cm}{s}\right]=\beta C d_{10}^2$	C is the Hazen coefficient in $1/[cm\cdot s]$ d_{10} is in cm	1	$100 < C < 150$
Kozeny (1953)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}\frac{n^3}{(1-n)^2}d_{10}^2$		8.3×10^{-4}	Large-grain sands
Kozeny-Carman (Carman 1937, 1956; Kozeny 1927, 1953)	$K\left[\frac{m}{s}\right]=\beta\frac{\rho g}{\mu}\frac{n^3}{(1-n)^2}d_{10}^2$		1/180	Silts, sands, and gravelly sands $d_{10} <$ 3 mm
Kruger (from Vukovic and Soro, 1992)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}\frac{n}{(1-n)^2}d_e^2$	g_i is the fractional percent weight retained on individual sieves	4.35×10^{-5}	$C > 5$
	$\frac{1}{d_e}=\sum_{i=1}^{n}\frac{\Delta g_i}{d_i}$	d_i is the mean grain diameter in mm of the corresponding fraction		Medium-grain sands
Krumbein and Monk (1943)	$K\left[\text{darcy}\right]=\beta \text{GM}_{\xi}^2 e^{-1.31\sigma_\phi}$	GM_ξ is the geometric mean in mm σ_ϕ is the phi standard deviation	760	
NAVFAC DM7 (1974; from Chesnaux et al. 2011)	$K\left[\frac{m}{s}\right]=\beta 10^{1.291e-0.6435}d_{10}^{10^{0.5504-0.2937e}}$	e is the void ratio	1	0.1 mm $< d_{10} <$ 2 mm
	$e=\frac{n}{1-n}$	d_{10} is in mm		$0.3 < e < 0.7$ $2 < C < 12$ $\frac{d_{10}}{d_5} > 1.4$
Pavchich (Pravedny 1966)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}d_{17}^2$		0.35	0.06 mm $< d_{17} <$ 1.5 mm
Sauerbrei (from Vukovic and Soro 1992)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}\frac{n^3}{(1-n)^2}d_{17}^2$		3.75×10^{-3}	Sand and sandy clay $d_{17} <$ 0.5 mm
Slichter (1899)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}n^{3.287}d_{10}^2$		0.01	0.01 mm $< d_{10} <$ 5 mm

Table 1
Continued

Method	Equation	Variable and Unit Definition	β	Use
Terzaghi (1925)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}\left(\frac{n-0.13}{\sqrt[3]{1-n}}\right)^2 d_{10}^2$		10.7×10^{-3} for smooth grains 6.1×10^{-3} for coarse grains	Large-grain sands
U.S. Bureau of Reclamation (from Vukovic and Soro 1992)	$K\left[\frac{m}{s}\right]=\beta\frac{g}{v}d_{20}^{2.3}$	d_{20} is in mm	4.8×10^{-4}	Medium-grain sands $C<5$
Zamarin (from Lu et al. 2012)	$K\left[\frac{m}{s}\right]=\beta_z\frac{g}{v}\frac{n^3}{(1-n)^2}d_e^2$	d_1 is the largest diameter of the finest fraction	8.2×10^{-3}	Large-grain sands
	$\frac{1}{d_e}=\frac{3}{2}\frac{\Delta g_1}{d_1}+\sum_{i=2}^{i=n}\Delta g_i\left(\frac{\ln\frac{d_i^g}{d_i^d}}{d_i^g-d_i^d}\right)$	Δg_1 is the weight of the material of the finest fraction in parts of the total weight d_i^g and d_i^d maximum and minimum grain diameters of the fraction, respectively Δg_i is the fraction weight in parts of the total weight		
Zunker (1932; from Lu et al. 2012)	$K\left[\frac{m}{s}\right]=\beta_z\frac{g}{v}\left(\frac{n}{1-n}\right)d_e^2$	d_1 is the largest diameter of the finest fraction	2.4×10^{-3} for uniform sand with smooth, rounded grains	Fine and medium-grain sands
	$\frac{1}{d_e}=\frac{3}{2}\frac{\Delta g_1}{d_1}+\sum_{i=2}^{i=n}\Delta g_i\left(\frac{d_i^g-d_i^d}{d_i^g d_i^d\ln\frac{d_i^g}{d_i^d}}\right)$	Δg_1 is the weight of the material of the finest fraction in parts of the total weight d_i^g and d_i^d maximum and minimum grain diameters of the fraction, respectively Δg_i is the fraction weight in parts of the total weight	1.4×10^{-3} for uniform composition with coarse grains 1.2×10^{-3} for nonuniform composition 0.7×10^{-3} for nonuniform compositions, clayey, with grains or irregular shape	

Table 2
Statistical Analysis of the Beach Depositional Environment Comparing the Measured and Estimated Hydraulic Conductivity Measurements for Each of the 20 Empirical Methods

Method (Beach)	R^2	−ln(p)	Mean [m/day]	Error [m/day]
Alyamani	0.12	7.35	41.86	175.22
Barr	0.3	17.99	69.70	227.80
Beyer	0.16	9.18	154.43	579.81
Chapuis	0.32	19.03	32.63	84.02
Fair and Hatch	0.3	17.99	96.23	314.50
Harleman	0.17	9.77	30.95	112.70
Hazen (original)	0.2	11.54	55.17	203.03
Hazen (modified)	0.17	9.77	0.43	1.56
Kozeny	0.25	14.57	4.52	16.62
Kozeny-Carman	0.17	9.77	27.55	100.31
Kruger	0.26	15.15	5.92	18.89
Krumbein	0.2	11.8	81.09	234.99
NAVFAC DM7	0.18	10.36	34.93	168.64
Pavchich	0.19	11.2	8.64	29.94
Sauerbrei	0.19	11.2	30.52	105.83
Slichter	0.23	13.41	16.26	59.75
Terzaghi	0.23	13.58	35.52	131.07
USBR	0.16	9.37	27.66	114.98
Zamarin	0.32	19.2	136.03	426.52
Zunker	0.26	15.18	199.10	605.99

Results

Comparison of Empirical Estimates vs. Measured Hydraulic Conductivities

Estimation of the hydraulic conductivity (temperature-normalized) using all 20 methods was applied to the grain-size distribution and measured porosity data for all 431 samples. Initially, no modification of the original equations was made. The estimated hydraulic conductivity values for all of the samples are presented in the supplemental data (Table S2).

A correlation analysis was then performed between the estimated and observed hydraulic conductivities, assuming a linear relation. The correlation coefficient R^2 and p-values were obtained for each general depositional group and then for the subgroups for each of the 20 methods analyzed. Data were used from all of the samples without removing any of the outliers. An example of this method for the beach sample group is provided in Table 2. Although the −ln(p) values showed a significant statistical correlation between the measured and estimated hydraulic conductivity values for each method, the errors were very high for 19 of the 20 methods with the modified Hazen method being the only exception. However, the Hazen (modified) method underestimates the hydraulic conductivity in most cases; the measured mean value for the beach samples is 27.15 m/day with a standard deviation of 14.03 m/day. Low R^2 values were found between most of the empirical estimation methods and the measured values within all of the other depositional environments and sub-environments.

The best correlated empirical methods were chosen based on the R^2 values for each classification of sediments with removal of outliers (outlier defined in methods). To facilitate the correlation of the empirical methods to the permeameter measurements for each depositional environment and sub-environment, a computer program was used to adjust the equation coefficients in order to obtain a linear relation with a unit slope. By doing so, a constant offset value was obtained for each method and a new beta coefficient was developed (Table 3). In addition, the program was used to estimate the expected error as two times the standard deviation of the residuals. When analyzing each data set, a range of effective diameters can be identified in which the linear relation holds best (d_{10} outlier definition). Poor correlations were found when samples that did not show the same physical properties (i.e., samples outside of the determined range) were included in the group. A number of samples were thus considered as outliers when obtaining the adjusted coefficients and the correlation coefficients. These outlier samples are marked in the supplemental data table (Table S2).

Hydraulic conductivity measurements made using a permeameter also have some error based on limitations of the method and the skill of the operator. Measurement of duplicate samples and a search of the cited literature suggest that the measurement error range is generally between 5 and 10% of the reported values for samples that contain low mud content. Poorly sorted samples that contain high mud percentages have a potentially higher measurement error. The statistical approach for analysis and improvement of the equations, used to estimate the hydraulic conductivity, take into consideration the measurement error. Samples with a high measurement error would likely become outliers and would be eliminated from the analysis. Samples with a small error in the range of 5 to 10% would fall between the two standard deviations and would not compromise the analysis.

Results of the statistical analyses between the measured and estimated hydraulic conductivity values with improvements to the equations, which include a revised beta value and an offset correction, are presented in Table 3. The modification of the beta coefficient, correction for the offset, and the grouping of the samples into major depositional groups and subgroups greatly improved the accuracy of the predicted hydraulic conductivity values. For example, an analysis of the grouped beach depositional environment shows that the estimated hydraulic conductivity values have a very poor correlation to the measured values shown as the blue line (Figure 1A). When only the methods that have the best correlation coefficients are applied, the estimated values of hydraulic conductivity correlate better with the measured values. However, there is still a high expected error between the estimated and measured values. Upon applying the modified beta values and offset corrections to the best correlated equations, there is considerable

Table 3
Recommended Empirical Equations Applied to Each Depositional Environment and Sub-Environment with New Beta Values, the Offset Correction, and Expected Error

Sedimentary Depositional Environment	Recommended Methods	R^2	−ln(p)	d_{10} [mm]	Mud [%]	Recommended β Coefficient	Offset [m/day]	Expected Error [m/day]	Samples Used for Analysis
Beach	Hazen (original)	0.55	36.6	$0 < d_{10} < 0.55$	<1.35	1.27×10^{-4}	17.71	13.61	88/92
	Zamarin	0.54	35.0			5.80×10^{-4}	18.68	13.40	
	Chapuis	0.54	36.2			0.88	16.60	13.15	
	Fair and Hatch	0.54	35.4			0.10	18.86	13.49	
	Barr	0.54	35.4			2.76×10^{-2}	18.86	13.49	
Beach (Siliciclastic)	Harleman	0.75	27.0	$0 < d_{10} < 0.51$	<1.49	3.00×10^{-4}	13.66	5.82	38/40
	Hazen (mod.)	0.75	27.0			35.10	13.66	5.82	
	Kozeny-Carman	0.75	27.0			2.90×10^{-3}	13.66	5.82	
	Beyer	0.72	25.3			1.00×10^{-4}	13.34	6.28	
	Hazen (original)	0.68	22.6			2.00×10^{-4}	14.06	6.75	
Beach (mixed)	Zunker	0.55	9.9	$0 < d_{10} < 0.66$	<0.61	0.72×10^{-4}	18.37	6.17	23/25
	Kruger	0.54	9.6			0.47×10^{-4}	18.47	6.33	
Beach (Carbonate)	Beyer	0.47	9.2	$0 < d_{10} < 0.50$	<1.61	2.51×10^{-5}	18.13	7.31	26/27
	Chapuis	0.47	9.3			9.78×10^{-1}	15.67	7.34	
	Harleman	0.47	9.3			2.52×10^{-5}	18.12	6.96	
	Hazen (original)	0.47	9.2			1.34×10^{-4}	18.11	7.32	
	Hazen (mod.)	0.47	9.3			14.83	18.12	6.96	
	Kozeny-Carman	0.47	9.3			1.90×10^{-3}	18.12	6.96	
Dune	Alyamani	0.63	34.6	$0 < d_{10} < 0.20$	<0.5	1.11×10^{-3}	5.77	4.92	68/71
	Beyer	0.62	34.3			1.47×10^{-4}	5.99	4.97	
	Harleman	0.62	33.7			8.13×10^{-4}	6.12	4.96	
	Hazen (original)	0.63	35.0			4.35×10^{-4}	4.86	4.90	
	Hazen (mod.)	0.62	33.7			89.82	6.12	4.96	
	Kozeny-Carman	0.62	33.7			4.90×10^{-3}	6.12	4.96	
Dune (Coast)	Krumbein	0.63	6.7	$0 < d_{10} < 0.31$	<1.84	273.24	14.08	6.82	13/14
	Harleman	0.61	6.4			4.24×10^{-4}	13.14	7.22	
	Hazen (mod.)	0.61	6.4			46.84	13.14	7.22	
	Kozeny-Carman	0.61	6.4			2.90×10^{-3}	13.14	7.22	
	Pavchich	0.61	6.4			8.29×10^{-1}	13.78	7.08	
	Sauerbrei	0.61	6.4			1.80×10^{-3}	13.78	7.08	
Dune (Interior)	Beyer	0.37	14.5	$0 < d_{10} < 0.10$	<6.83	2.77×10^{-4}	2.08	3.42	56/57
	Harleman	0.38	14.7			1.60×10^{-3}	1.60	3.38	
	Hazen (mod.)	0.38	14.7			181.95	1.60	3.38	
	Kozeny-Carman	0.38	14.7			9.80×10^{-3}	1.60	3.38	

Table 3
Continued

Sedimentary Depositional Environment	Recommended Methods	R^2	−ln(p)	d_{10} [mm]	Mud [%]	Recommended β Coefficient	Offset [m/day]	Expected Error [m/day]	Samples Used for Analysis
Offshore	Kruger	0.64	114.1	$0< d_{10} <0.28$	<7.1	1.49×10^{-4}	9.54	10.12	218/228
	Zunker	0.64	113.9			3.18×10^{-4}	7.88	9.84	
Offshore (Carbonate, 2 to 15% mud)	Barr	0.39	12.5	$0< d_{10} <0.15$	<12.82	6.40×10^{-2}	4.59	5.61	46/46
	Fair and Hatch	0.39	12.5			2.32×10^{-4}	4.59	5.61	
	Kruger	0.40	12.9			1.87×10^{-4}	3.76	5.68	
	Zunker	0.36	11.3			4.39×10^{-4}	2.42	5.71	
Offshore (Carbonate, less than 2% mud)	Kruger	0.62	44.4	$0< d_{10} <0.29$	<1.69	1.42×10^{-4}	12.51	11.07	89/90
	Zunker	0.63	45.3			2.84×10^{-4}	10.25	10.91	
Offshore (Mixed)	Pavchich	0.55	18.8	$0< d_{10} <0.28$	<1.54	7.75×10^{-1}	10.28	5.88	45/47
	Sauerbrei	0.55	18.8			2.20×10^{-3}	10.28	5.88	
Offshore (Siliciclastic)	Beyer	0.57	19.0	$0< d_{10} <0.21$	<4.34	1.25×10^{-4}	8.12	2.17	43/45
	Harleman	0.50	16.0			5.95×10^{-4}	8.70	2.19	
	Hazen (original)	0.54	17.8			4.44×10^{-4}	5.81	2.24	
	Hazen (mod.)	0.50	16.0			65.71	8.70	2.19	
	Kozeny-Carman	0.50	16.0			3.10×10^{-3}	8.70	2.19	
River	Beyer	0.07	2.3	$0< d_{10} <0.75$	<1.81	1.30×10^{-5}	39.10	23.07	37/40
	Harleman	0.08	2.4			6.83×10^{-5}	38.67	22.95	
	Hazen (mod.)	0.08	2.4			7.54	38.67	22.95	
	Kozeny-Carman	0.08	2.4			8.14×10^{-4}	38.67	22.95	
River (Wadi)	Beyer	0.66	10.8	$0< d_{10} <0.51$	<1.97	1.80×10^{-4}	−8.32	18.51	19/20
	Harleman	0.64	10.1			8.74×10^{-4}	−9.42	18.43	
	Hazen (original)	0.66	10.6			5.04×10^{-4}	−1.51	19.29	
	Hazen (mod.)	0.64	10.1			96.59	−9.42	18.43	
	Kozeny-Carman	0.64	10.1			1.52×10^{-2}	−9.42	18.43	
River (Perennial)	Krumbein	0.09	1.58	$0< d_{10} <0.92$	<1.67	4.93	43.07	12.96	19/20

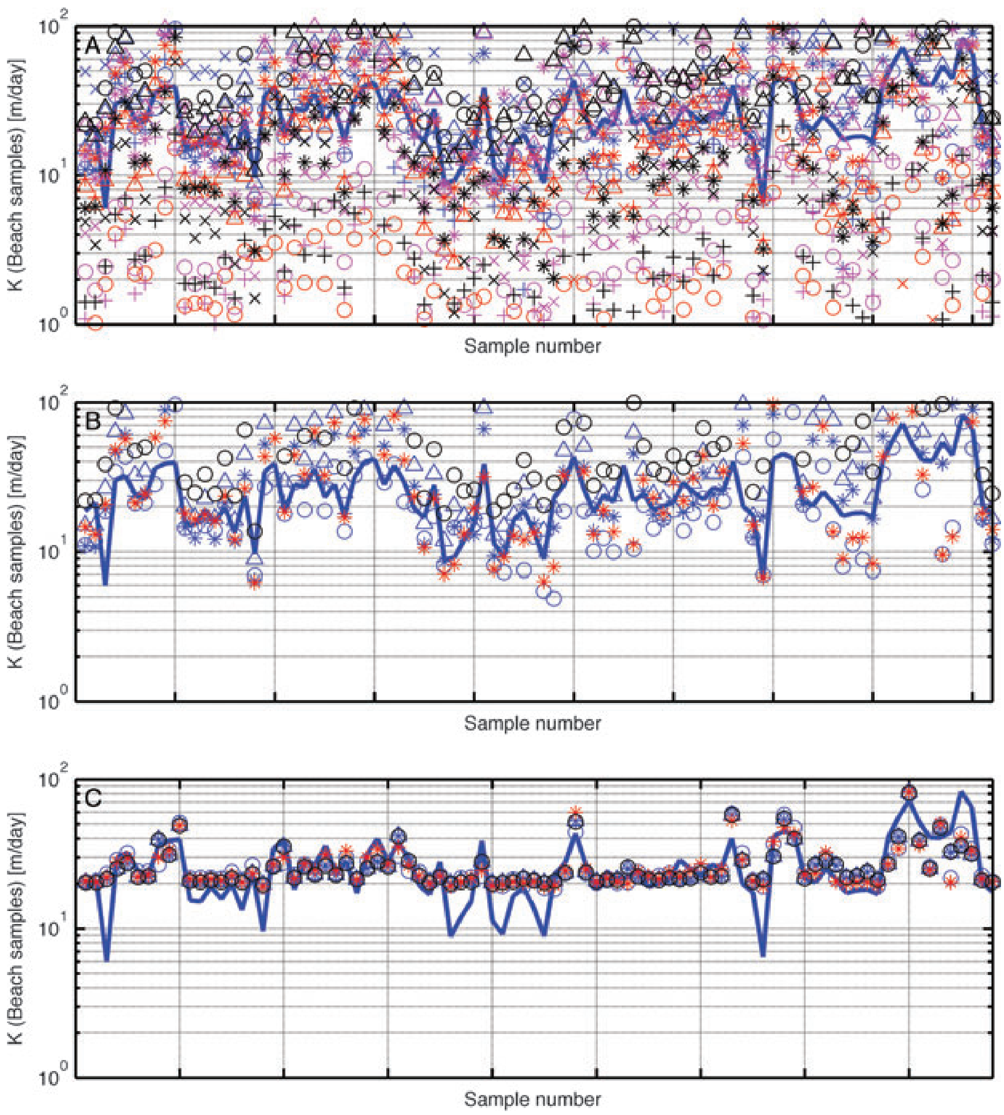

Figure 1. Estimated hydraulic conductivity using the 20 empirical methods for all beach samples compared to measured hydraulic conductivity measurements (blue line) (A). Comparison of the estimated hydraulic conductivities to the measured values for the five best empirical methods without changes to the equations (see Table 3) (B). Comparison of the estimated hydraulic conductivities to the measured values for the five best empirical methods with modifications to the equations (C).

improvement to the estimated hydraulic conductivity (Figure 1C).

Application of the most effective empirical methods applied to samples classified into sub-environments further improved the accuracy of the predicted hydraulic conductivity (Table 3). In addition, specific limitations on the modified methods were established based on the range in d_{10} and mud percentage which were found to have a significant influence on the predicted and measured hydraulic conductivity values and the associated errors. Plots of calculated vs. measured hydraulic conductivity measurements for all of the environments and sub-environments with the outlier points removed for selected methods are shown in Figures 2 and 3. The R^2 and p-values are shown in all of the plots which are linear in nature. Also, it should be noted that the best fit line does not intercept the y-axis at zero. This occurs because the very low hydraulic conductivity values are controlled by d_{10} and mud percentages that fall outside of the statistical validity of the methods applied, therefore making the correlation nonlinear in this part of the curve.

All of the modified equations provide a reasonable statistical correlation between the estimated and measured hydraulic conductivity values with the exception of the perennial stream sample set. Both the R^2 and p-values indicate a poor correlation. However, the modified empirical methods still show an improvement compared to the unmodified methods.

Correlation of Measured Hydraulic Conductivity with Statistical Moments

Statistical analyses were conducted on the relationships between measured hydraulic conductivity values and the corresponding statistical moments (e.g., mean, standard deviation, skewness, kurtosis). A few significant correlations were found with the measured hydraulic

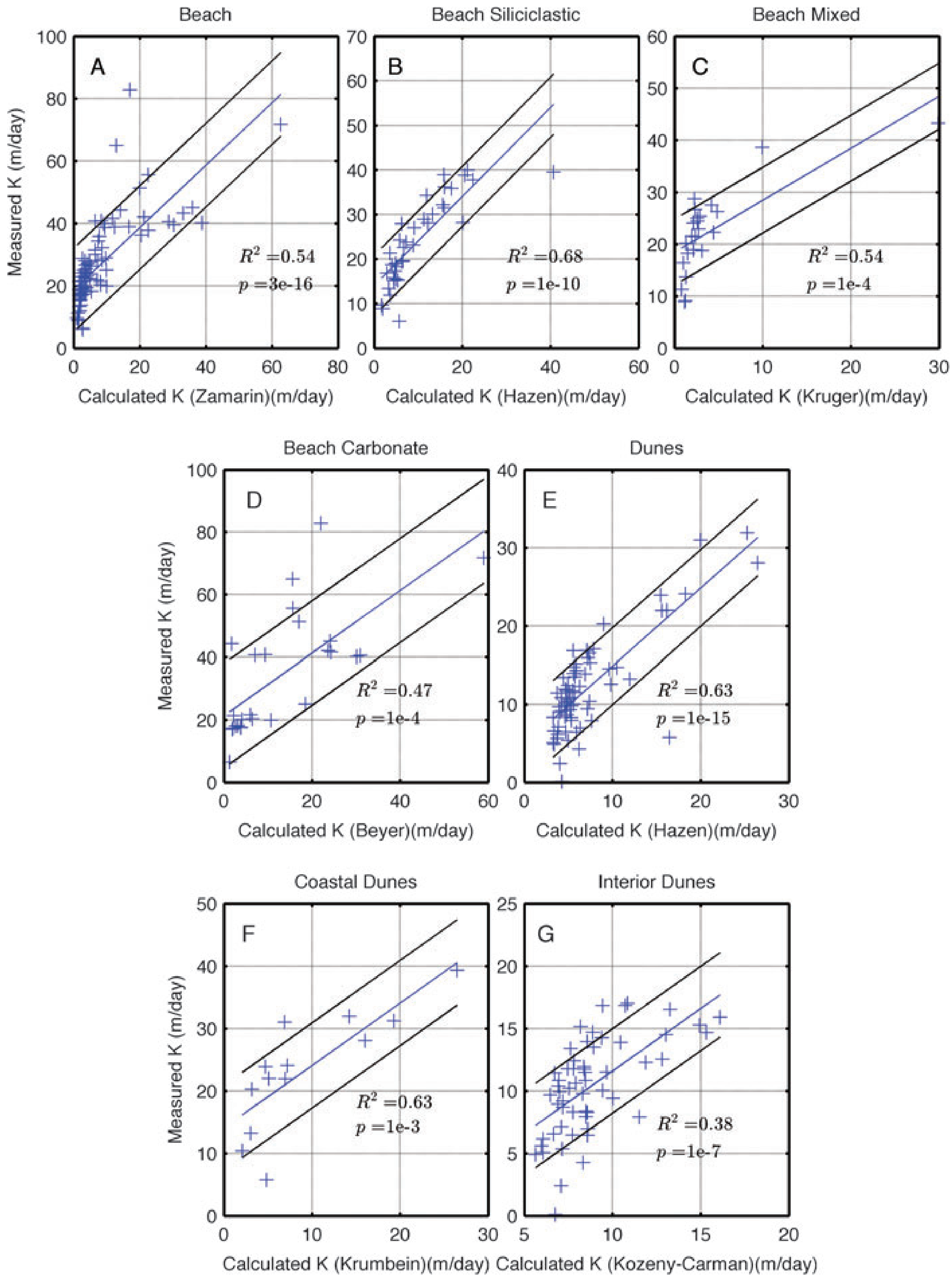

Figure 2. Plots of calculated vs. measured hydraulic conductivity for some of the best methods (high R^2) for total beach sediments (A), siliciclastic beach sediments (B), mixed siliciclastic and carbonate beach sediments (C), carbonate beach sediments (D), dunes (E), coastal dunes (F), and interior dunes (G). The central line is the best fit and the other two lines show the boundaries of the two standard deviations.

conductivity of siliciclastic beach sands being related to mean grain diameter in phi units ($R^2 = 0.4$, $p = 2 \times 10^{-5}$) (Figure 4). For coastal dunes, correlations were found between the measured hydraulic conductivity and mean grain diameter in phi units ($R^2 = 0.67$, $p = 1 \times 10^{-3}$), standard deviation in phi units ($R^2 = 0.61$, $p = 2 \times 10^{-3}$), and mean grain diameter in millimeter ($R^2 = 0.33$, $p = 1 \times 10^{-2}$). However, no new empirical equations to estimate hydraulic conductivity from these relationships could be developed that are more accurate than those

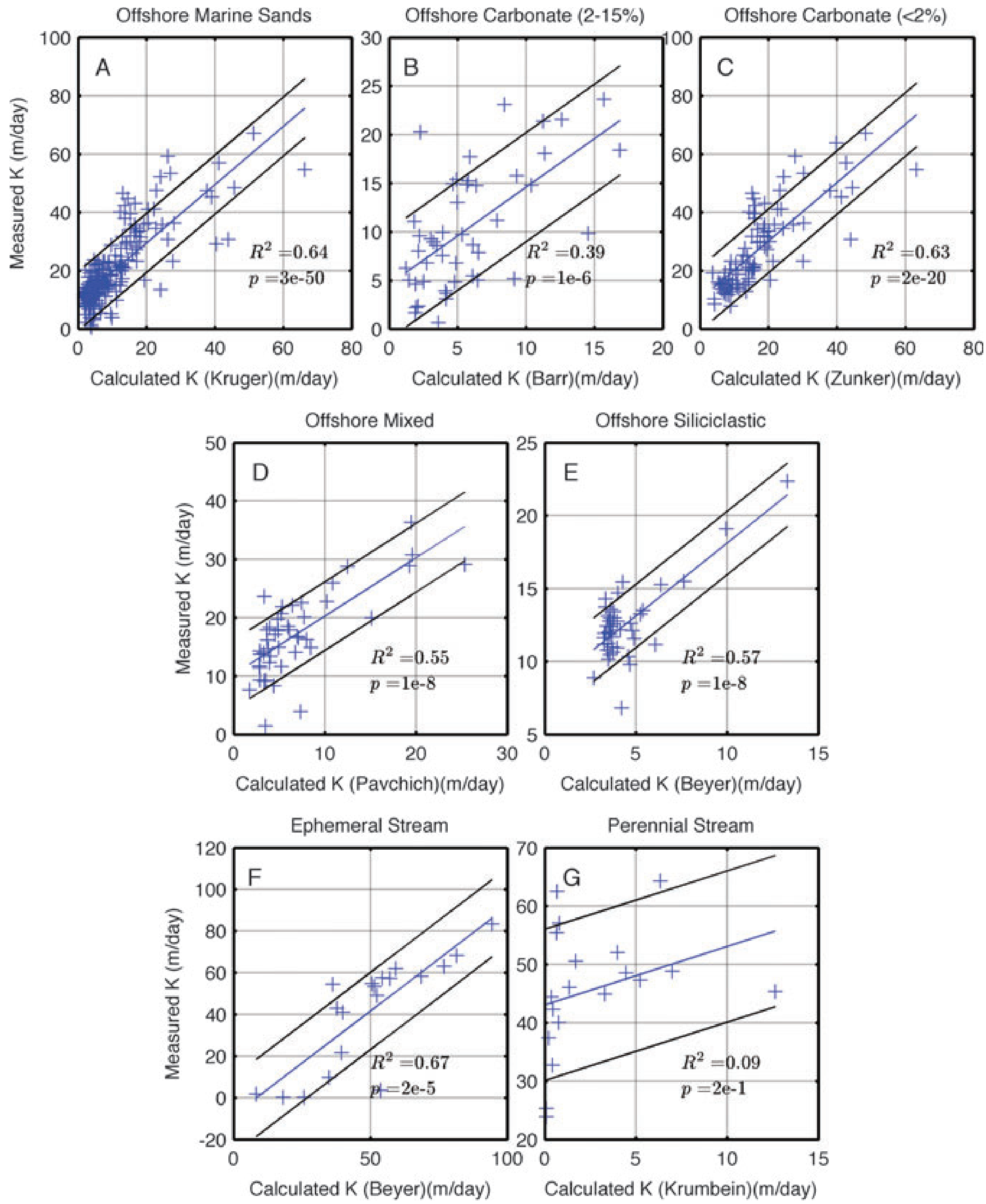

Figure 3. Plots of calculated vs. measured hydraulic conductivity for some of the best methods (high R^2) for total offshore sediments (A), carbonate offshore sediments with mud between 2 and 15% (B), carbonate offshore sediments with mud <2% (C), mixed offshore siliciclastic and carbonate beach sediments (D), offshore siliciclastic sediments (E), ephemeral streams (F), and perennial streams (G). The central line is the best fit and the other two lines show the boundaries of the two standard deviations.

given in Table 3. The statistical moments did play a role in the Krumbein empirical equation statistical relationship for analysis of coastal dunes and perennial rivers (Krumbein uses a statistical moment in his empirical equation).

Discussion

Grain-size distribution data have been commonly used to estimate the hydraulic conductivity of sediments using a variety of different empirical equations. From this analysis, it is clear that each of the empirical equations yields different hydraulic conductivity estimates compared to measured values using a permeameter for a given sediment sample with greatly varying accuracy. By grouping of the sediment samples into specific depositional environments and sub-environments based on some physical properties, such as mineralogy and mud percentage, the statistical accuracy of all 20 methods applied to each group was assessed. Specific empirical

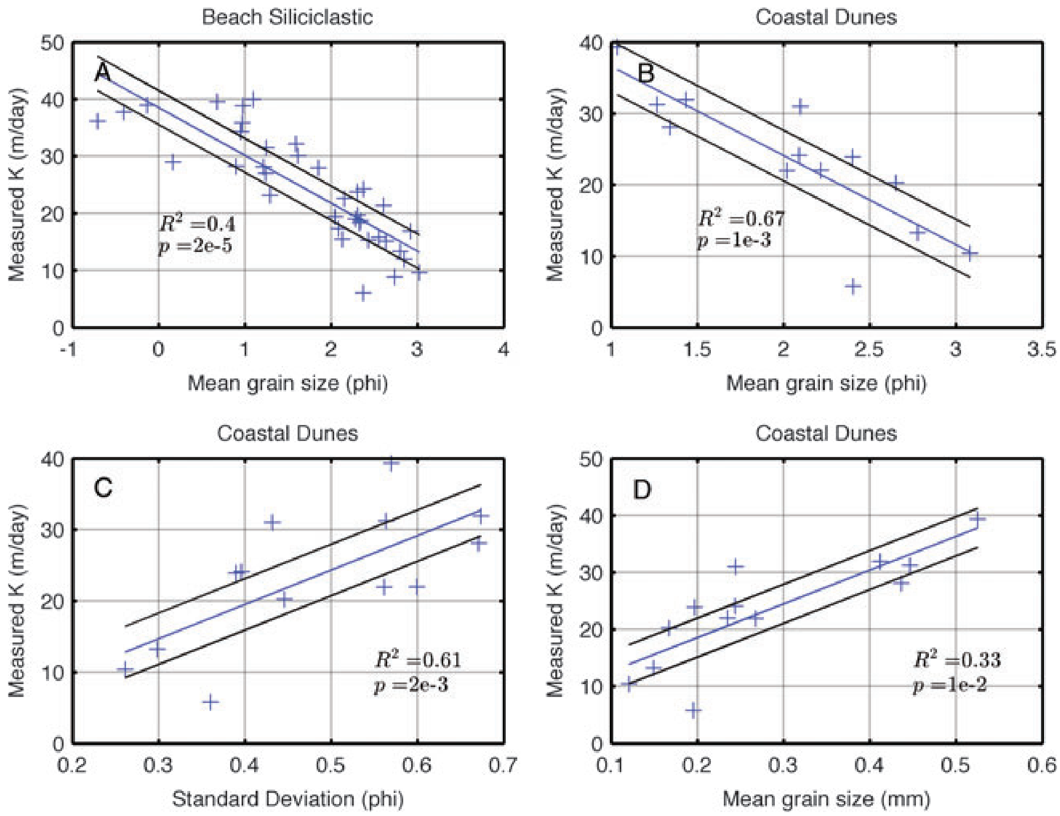

Figure 4. Measured hydraulic conductivity vs. selected statistical moments for siliciclastic beach sediments (A), and coastal dune sediments (B to D). The central line is the best fit and the other two lines show the boundaries of the two standard deviations.

equations were shown to predict the hydraulic conductivity more accurately than others for a given environment or sub-environment, but all of these equations still had a high degree of predictive error. By adjusting the beta values to create a linear relationship between the measured and estimated hydraulic conductivity values for each equation and using a regression analysis, a correction offset was found to improve the empirical equation estimates. The modified equations greatly improve the accuracy of hydraulic conductivity estimates for the beach, dune, and offshore sub-environments. The empirical method estimations of hydraulic conductivity for rivers were improved to a lesser degree.

The hydraulic conductivity of specific samples cannot be accurately estimated by any of the empirical methods and must be defined as outliers. Within the context of this analysis, outliers were defined as samples that contain a d_{10} value or mud percentage that deviate more than three standard deviations outside of the group mean value. Elimination of outliers within the depositional environments and sub-environments improves the overall statistical accuracy for the data set.

Because the program used to obtain comparable relationships (in Appendix S1) modifies the beta coefficients of the empirical equations, some of the methods become nearly identical or very similar. For example, the Harleman and Hazen (modified) equations only differ by the factor $\rho g/\mu$. Because the sample measured and estimated hydraulic conductivities are normalized to a specific temperature, this factor remains constant. Moreover, when the program is used to modify the beta coefficients, these two methods yield identical hydraulic conductivity estimates. The Kozeny-Carman equation differs from these two methods by a function that is proportional to the square of the porosity. If the variations in porosity among a particular data set are not significant, then the Kozeny-Carman equation will yield hydraulic conductivity estimates that are very similar to the Harleman and Hazen (modified) methods after adjustments to the coefficients. Therefore, similar empirical methods may yield the same correlation coefficients for a particular set of samples. The improved hydraulic conductivity estimates using the modified empirical methods applied to the samples are given in Table S3.

Conclusions

A statistical assessment of 20 empirical equations used to estimate the hydraulic conductivity of 431 unlithified sediments samples was made by comparing measured vales to the estimated values. All 20 methods poorly predicted the measured values with errors ranging over 500%. To improve the hydraulic conductivity estimates, sediment samples were first divided into depositional environments and sub-environments based lithology and mud percentage. Next, the statistical significance of the estimated hydraulic conductivity values for the 20 empirical equations was assessed for each sediment classification. A linear relationship between the measured and the estimated values was established using a computer program to adjust the beta coefficients of the best correlated empirical methods. Outlier samples were eliminated by defining a range of acceptable d_{10} and mud percentage values for each method (three standard deviations from the group mean). The offset of the linear relationship was determined for each group to improve the predictive accuracy of the selected methods. The empirical equations producing the most accurate predictions were then modified by using the adjusted beta values and applying the offset.

Classification of unlithified sediment into subgroups used on lithology and mud percent generally improves accuracy of the estimated hydraulic conductivity using the modified empirical equations. For example, the expected error for the beach environment samples as a group is 13.5 m/day, while the subgroups of siliciclastic, mixed, and carbonate sediment produce expected errors of 6, 6.2, and 7.1 m/day respectively. The combined dune group shows an expected error of 5 m/day, but the coastal dunes subgroup has an expected error of 7.1 m/day and the interior dunes has a much lower expected error at 3.4 m/day. The offshore sediments as a total environment have an expected error of 10 m/day with considerably lower expected errors for the muddy carbonate (2 to 15%), mixed siliciclastic/carbonate, and siliciclastic subenvironments at 5.6, 5.9, and 2.2 m/day, respectively. The expected error of the low mud carbonate (2%) subenvironment was higher at 11 m/day. River sediments had a poor statistical correlation to all of the empirical methods with an expected error for the improved methods at 23 m/day. Separation of the rivers into wadis and perennial rivers did show some improvement in expected error at 18.5 and 13.0 m/day, respectively. However, the perennial river relationship may not be truly statistically significant because of the small number of samples.

Analysis of the relationships between measured hydraulic conductivity values and the grain-size distribution statistical moments, mean grain diameter, standard deviation, skewness, and kurtosis, were generally poor with a few exceptions. No new empirical equations could be developed that accurately predict the hydraulic conductivity from these relationships.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Description of computer programs with user instructions.

Table S1. Sediment Samples Analyzed for Hydraulic Conductivity, Laboratory Porosity, and Grain-Size Distribution

Table S2. Measured and estimated hydraulic conductivities (existing empirical equations), statistical moments, d10, d17, d20, d50, d60, and mud percent for all samples.

Table S3. Measured and estimated hydraulic conductivities (new equations), statistical moments, d10, d17, d20, d50, d60, and mud percent for all samples.

MULTI H_K program in EXCEL

Grain-size hydraulics program in MATLAB®

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